

## **Trigonometric Identities**

Here is a list of many of the identities from trigonometry. These identities may be used to verify or establish other identities.

Reciprocal Identities	Ratio Identities
$\cot \cot \theta = \frac{1}{\tan \tan \theta}$	$\tan \tan \theta = \frac{\sin \sin \theta}{\cos \cos \theta}$
$\sec \sec \theta = \frac{1}{\cos \cos \theta}$	$\cot \cot \theta = \frac{\cos \cos \theta}{\sin \sin \theta}$
$\csc \csc \theta = \frac{1}{\sin \sin \theta}$	
Negative-Angle Identities	
$\sin \sin (-\theta) = -\sin \sin \theta \cos \cos \theta$	$(-\theta) = \cos \cos \theta$
$\tan \tan (-\theta) = -\tan \tan \theta$	
$\csc \csc (-\theta) = -\csc \csc \theta \qquad \qquad \sec \sec \theta$	$(-\theta) = \sec \sec \theta$
$\cot \cot (-\theta) = -\cot \cot \theta$	
Co-function Identities	Pythagorean Identities
$\sin \sin (90^\circ - \theta) = \cos \cos \theta$ $sec(90^\circ - \theta) = \csc csc \theta$	$\theta + \theta = 1$
$\cos \cos (90^\circ - \theta) = \sin \sin \theta$	$\theta + 1 = \theta$
$csc(90^\circ - \theta) = \sec sec \ \theta$	$1 + \theta = \theta$
$\tan \tan \left(90^\circ - \theta\right) = \cot \cot \theta$	
$\cot \cot (90^\circ - \theta) = \tan \tan \theta$	
Sum & Difference Identities	
$cos (\alpha + \beta) = cos \alpha cos \beta - sin \alpha sin \beta$ $cos (\alpha - \beta) = cos acos \beta + sin \alpha sin \beta$	
$sin (\alpha + \beta) = sin \alpha cos \beta + cos \alpha sin \beta$ $sin (\alpha - \beta) = sin \alpha cos \beta - cos \alpha sin \beta$	
$tan (\alpha + \beta) = \frac{tan \alpha + tan \beta}{1 - tan \alpha tan \beta} \qquad tan (\alpha - \beta) = \frac{tan \alpha - tan \beta}{1 + tan \alpha tan \beta}$	
Double-Angle Identities	
$sin(2\alpha) = 2sin \alpha cos \alpha$	
$\cos \cos (2\alpha) = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2 \alpha - 1 = 1 - 2\sin^2 \alpha$	
$\tan\left(2\alpha\right) = \frac{2\tan\alpha}{1-\tan^2\alpha}$	
Half-Angle Identities	
$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1+\cos\alpha}{2}} \qquad \tan\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$	
Product to Sum Identities	Sum to Product Identities
$sin \alpha cos \beta = \frac{sin (\alpha + \beta) + sin(\alpha - \beta)}{2}$	$\sin \alpha + \sin \beta = 2 \sin \left(\frac{a+b}{2}\right) \cos \left(\frac{\alpha-\beta}{2}\right)$
$\sin \alpha \sin \beta = \frac{\cos (\alpha - \beta) - \cos(\alpha + \beta)}{2}$	$\sin \alpha - \sin \beta = 2 \cos \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)$
$\cos \alpha \cos \beta = \frac{\cos (\alpha + \beta) + \cos(\alpha - \beta)}{2}$	$\cos \alpha + \cos \beta = 2 \cos \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)$
$\frac{1}{2}$	$\cos \alpha - \cos \beta = -2 \sin \left(\frac{\alpha+\beta}{2}\right) \sin \left(\frac{\alpha-\beta}{2}\right)$



